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Multivariable Control System Design for the Robust control of Twin Rotor MIMO System using Quantitative Feedback Theory

Ambili Mohan[1] and Gylson Thomas[2]
[1]Muthoot Institute of Technology & Science/Department of Electrical Engineering, Ernakulum, India
Email: ambilimohan@mgits.ac.in
[2]Muthoot Institute of Technology & Science/Department of Electrical Engineering, Ernakulum, India
Email: gylsonthomas@mgits.ac.in

***Abstract*—This paper focuses on design of multivariable controller for twin rotor MIMO system (TRMS), a two input two output system with large plant uncertainty using quantitative feedback theory (QFT) methodology. The various QFT based controller for TRMS, reported earlier, consider two approaches; cross coupling effects are neglected in one of them while the other approach employs a decoupler to take into account of the coupling effects. The objective of this paper is to propose a QFT based diagonal robust controller by using the multivariable structure of the TRMS, so that the need for a separate decoupling is completely avoided. Thus the main advantage of this strategy is that the design procedure is made simpler to meet the time domain tracking specifications and robust stability requirement in positioning the pitch and yaw angles of the TRMS.**

***Index Terms*— Twin Rotor MIMO System, Quantitative Feedback Theory, Diagonal controller.**

I. Introduction

The Twin rotor MIMO (multi input multi output system) system is a laboratory sized helicopter model normally used as a benchmark example for a multivariable control system. TRMS contains in itself a collection of characteristics namely MIMO structure, nonlinear nature and cross coupling. Thus it serves as a good platform to investigate the range of effectiveness of new control strategies. The power of Quantitative Feedback Theory (QFT) in dealing with system uncertainties is exploited in this paper to design a controller for the TRMS in achieving the tracking of pitch and yaw angles.

The system mechanical characteristics and also the nonlinear model of the system are well explained in the user manual [1]. Several investigations have been conducted in developing robust control technique for the TRMS. “Ref [2]” describes a robust control strategy used in TRMS. The method involves decoupling the TRMS into two SISO subsystems for applying the control strategy which is basically developed for SISO system. A main drawback is that the final parameter values had to be largely tuned in real time implementation. Twin Rotor MIMO system is decoupled and fuzzy T-S modeled in [3]. Here the nonlinear TRMS is quasi linearized into several LTI subsystems with respect to different operating points. With the parallel distributed fuzzy LQR controller yaw and pitch angle are positioned at desired values and it is robust

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to disturbance. Robust controller for TRMS using QFT had been designed in [4]. Here the cross coupling effects are neglected and also the controllers are described by improper transfer functions. The contribution of [5] is to present the QFT design for non-parametric uncertainty of unstructured type model. Here plant modeling is only for main pitch path, i.e. yaw path is not considered. The objective of [6] is to design a robust controller based on QFT, for achieving tracking performance in the pitch and yaw channels by designing a decoupler to compensate for the effect of cross coupling dynamics. Controllers have also been designed for both the channels by neglecting the cross coupling effects and they are found to be described by proper transfer functions; thus a comparison is made with respect to the controller form in [4]. In [7] the multivariable controller for quadruple tank process has been designed, with large plant uncertainty using QFT methodology. The main focus of the present paper is to propose a robust controller based on QFT, for achieving tracking performance in the pitch and yaw channels. The strategy is also devoid of the problems associated with the nonsuitability of effectiveness of the decoupler in the presence of plant parameter variations.

This paper is organized as follows. The dynamic model of TRMS is presented in section II . Design of the decoupler is described in section III. QFT design procedure and simulation results are explained in section IV and section V involves conclusion.

II. MODEL DESCRIPTION OF TRMS

As shown in “Fig. 1,” TRMS mechanical unit consists of a beam pivoted on its base in such a way that it can rotate freely both in its horizontal and vertical planes. At both ends of the beam there are two propellers driven by DC motor. The control inputs are the supply voltages of the DC motors. TRMS system has main and tail rotors for generating vertical and horizontal propeller thrust. The main rotor produces a lifting force allowing the beam to rise vertically, making a rotation around the pitch axis. While the tail rotor is used to make the beam turn left or right around the yaw axis. The TRMS system has two degree of freedom (2-DOF), the pitch and the yaw and the only mode of flight is hovering.

The measured signals are position of beam in space ie two position angles. The positions of beam are measured by means of incremental encoders. Thus there are two inputs - horizontal and vertical voltages, and two outputs - pitch and yaw angles for the TRMS.

A. System mathematical Model

For the derivation of the approximate mathematical model, the parameters in the TRMS are first presented as follows, $l_m(l_t)$ is the main (tail) length of the beam, $J_{mr}(J_{tr})$ is the moment of inertia for the main (tail) propeller subsystem, $T_{mr}(T_{tr})$ is the time constant of the main (tail) motor- propeller system, $\omega_m(\omega_t)$ is the rotational speeds of the main (tail) DC-motor, $k_v(k_h)$ is the friction constant of the main (tail) propeller subsystem, S_f is balanced scale, *D*, *E* and *G* are constants, $\alpha_v(\alpha_h)$ is the pitch (yaw) angle, $\Omega_v(\Omega_h)$ the angular velocity around the vertical (horizontal) axis, $u_{vv}(u_{hh})$ is the output of main (tail) DC- motor, $u_v(u_h)$ is the control input for the main (tail) DC-motor, F_v (F_h) is the propulsive force to move the joined beam in the vertical (horizontal) direction. $F_v(F_h)$ is described by nonlinear function of angular velocity $\omega_m(\omega_t)$ and *g* is gravitational acceleration.

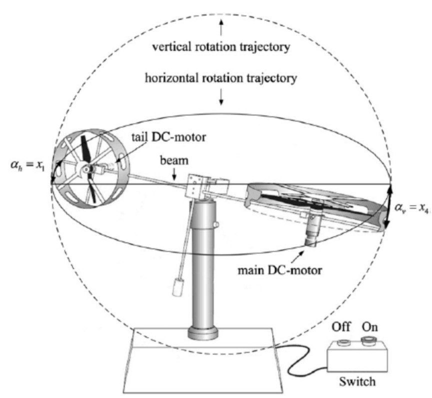


Figure 1. Twin rotor multi input multi output system (TRMS)

Modifying the dynamical equations of TRMS [1] and defining the state variables as $x_1 = \alpha_v$, $x_2 = \Omega_v$, $x_3 = u_{vv}$, $x_4 = \alpha_h$, $x_5 = \Omega_h$, $x_6 = u_{hh}$, the state equations are finally obtained as:

$$\dot{x}_1 = x_2 \tag{1}$$

$$\dot{x}_2 = 9.1\begin{bmatrix} l_m S_f F_v(\omega_m) - K_v x_2 - g(0.0099 \cos x_1 + 0.0168 \sin x_1) \\ -0.0252 x_5^2 \sin 2x_1 + J_{tr}\,\dot{\omega}_t (u_h - x_6)/T_{tr} \end{bmatrix} \tag{2}$$

$$\dot{x}_3 = (u_v - x_3)/T_{mr} \tag{3}$$

$$\dot{x}_4 = x_5 \tag{4}$$

$$\dot{x}_5 = \frac{l_t S_f F_h(\omega_t)\cos x_1 - x_5 k_h}{J_h} - \frac{x_5 x_2 (E-D)\sin 2x_1}{J_h} - \frac{J_{mr}\omega_m x_2 \sin x_1}{J_h} + \frac{J_{mr}(u_v - x_3)/T_{mr}\,\dot{\omega}_m \cos x_1}{J_h} \tag{5}$$

$$\dot{x}_6 = (u_h - x_6)/T_{tr} \tag{6}$$

Since the dynamic characteristics of TRMS are very complex in nature, it would be convenient to design a controller, with the system separated in to vertical and horizontal subsystems with states as $x_v = [x_1\ x_2\ x_3]$ and $x_h = [x_4\ x_5\ x_6]$. The angular velocity of main (tail) rotor is a nonlinear function of output voltage and can be approximated by:

$$\omega_m = 90.99u_{vv}^6 + 599.73u_{vv}^5 - 129.26u_{vv}^4 - 1238.64u_{vv}^3 + 63.45u_{vv}^2 + 1283u_{vv} \tag{7}$$

$$\omega_t = 2020u_{hh}^5 - 194.68u_{hh}^4 - 4283.15u_{hh}^3 + 262.27u_{hh}^2 + 3796.83u_{hh} \tag{8}$$

Propulsive force can be described approximately by a nonlinear function of the angular velocity as follows:

$$F_v = -3.48*10^{-12}\omega_m^5 + 1.09*10^{-9}\omega_m^4 + 4.123*10^{-6}\omega_m^3 - 1.632*10^{-4}\omega_m^2 + 9.544*10^{-2}\omega_m \tag{9}$$

$$F_h = -3*10^{-14}\omega_t^5 - 1.595*10^{-11}\omega_t^4 + 2.511*10^{-7}\omega_t^3 - 1.808*10^{-4}\omega_t^2 + 8.01*10^{-2}\omega_t \tag{10}$$

To obtain a suitable linearized model with more accuracy, highly nonlinear functions in the state equations are simplified as the propositional combination of linear functions [3]. Now Using the dynamic equations "(1-6)" and by Jacobian linearization the system equations are written in the form *Y(s)=G(s)U(s),* where

$$G(s)=\begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \qquad (11)$$

Using this model in "(11)," QFT based diagonal controllers have been designed [section IV] for the multi-input multi-output system.

III. QFT DESIGN BASIC CONCEPTS

Quantitative feedback theory (QFT), developed by Isaac Horowitz [8] is a frequency domain technique utilizing the Nichols chart in order to achieve a desired robust design over a specified region of plant uncertainty. In this paper QFT is used to design a controller (i) to solve the problem of parametric uncertainties in the model, and (ii) to enhance the robust stability and tracking performance of the TRMS.

The QFT design procedure involves four basic steps: generation of plant templates, computation of QFT bounds, design of the controller (loop shaping) and the design of prefilter. Given the plant templates, QFT converts closed loop magnitude specifications into magnitude constraints on a nominal open-loop function (QFT bounds). A nominal open-loop function *L(s)*, defined as the product of the controller transfer function and the plant transfer function, is then designed to satisfy simultaneously the plant template constraints as well as to achieve nominal closed-loop stability (loop shaping). Prefilter *F(s)*, is required to transfer the variations obtained with the design to the zone defined by the tolerances $T_{RL}(s)$ and $T_{RU}(s)$as in "(14),"and "(15),". A frequency array is selected for computing templates and bounds. The two degree of freedom feedback system configuration is shown in fig 2. G(s) and F(s) are controller and prefiter transfer functions respectively and P(s) is the plant transfer function. L(s) = G(s) P(s). The feedback system of interest are depicted mathematically by

$$y = Pu; \qquad u = G(Fr - y)$$

$$T = (I + PG)^{-1} PGF; \qquad [P^{-1} + G]T = GF \qquad (12)$$

A. Bound Calculations

For the vertical and horizontal systems obtained without decoupler and with decoupler, the same bounds are considered in this paper. The tracking specification defines the acceptable range of variation of the closed loop tracking response of the system due to uncertainty and disturbances. It is generally defined in the time-domain, but normally transformed into the frequency domain, as expressed by the following inequality.

$$T_{RL}(j\omega) \le T_R(j\omega) \le T_{RU}(j\omega) \qquad (13)$$

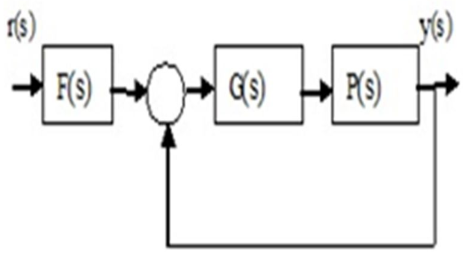
The function $T_R(s)$denotes the closed-loop transfer functions and $T_{RL}(s)$and $T_{RU}(s)$ are the equivalent transfer functions of the lower and upper tracking bounds. An overshoot of 20% and settling time of 5 seconds are specified, and the equivalent bound transfer functions are constructed as follows,

$$T_{RU}(s) = \frac{s+9}{s^2+3s+9} \qquad (14)$$

$$T_{RL}(s) = \frac{20}{s^3+11s^2+30s+20} \qquad (15)$$

The robust stability specification for a gain margin of 2 dB and phase margin of 45^0 [8] is defined as

$$\left|\frac{1}{1+L}\right| \le 1.2 \qquad (16)$$



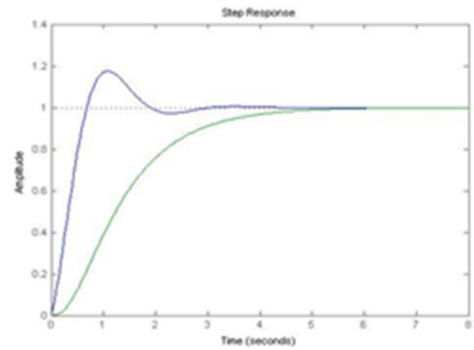


Figure 2. Feedback control system configuration for QFT

Figure 3. Bounds in time domain

"Fig.3," shows step response for upper and lower tracking bounds corresponding to "(14) and (15),".

IV. QFT METHOD FOR MIMO PLANT

The multivariable version of two degree of freedom controller structure for the TRMS is shown in "Fig. 4,". The basic idea is to break the design process down into a series of stages. A Solution to the original problem is then simply a combination of the solutions obtained at each stage.
Its transfer function matrix is then obtained by Y(s) = P(s) R(s), where

$$P(s)=\begin{bmatrix} p_{11}(s) & p_{12}(s) \\ p_{21}(s) & p_{22}(s) \end{bmatrix} \quad and \quad T(s)=\begin{bmatrix} t_{11}(s) & t_{12}(s) \\ t_{21}(s) & t_{22}(s) \end{bmatrix} \tag{17}$$

Controller and prefilter are chosen as diagonal matrices. The four transfer functions are derived as

$$\begin{aligned} &for\ input\ r_1: \ t_{11}=\frac{g_1 f_1 - t_{21}\pi_{12}}{\pi_{11}+g_1} \qquad t_{12}=\frac{-t_{11}\pi_{21}}{\pi_{22}+g_2} \\ &for\ input\ r_2: \ t_{12}=\frac{-t_{22}\pi_{12}}{\pi_{11}+g_1} \qquad t_{12}=\frac{g_2 f_2 - t_{11}\pi_{21}}{\pi_{22}+g_2} \end{aligned} \tag{18}$$

Where $P^{-1}=[\pi_{ij}]$

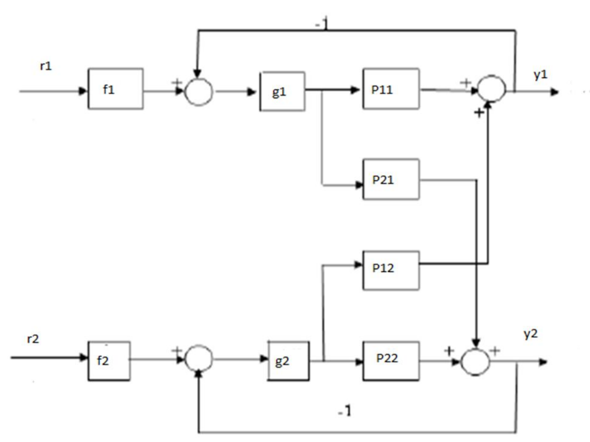


Figure 4. Controller structure for a 2x2 MIMO system

These four transfer functions represent the resulting four equivalent single-loop multi input single output (MISO) control systems. There are two inputs and one output. One input represents the system input (desired input); the other represents the cross-coupling effects from all other loop outputs. The object of the design is to have each loop track its desired reference, while minimizing the outputs due to the cross-coupling effects. It follows that only the two systems on the diagonal line have both input tracking and disturbance-rejection requirements. The other two systems have only disturbance-rejection requirement. In summary, the design procedure based on direct MIMOQFT approach consists of the following steps:

a) Determine the transfer function matrix of the plant dynamics.
b) Define the transfer functions of the equivalent MISO subsystems.
c) Robust controller design for the subsystems.
d) Integration of the design results to obtain the overall control system.

Next step is the design of loops. Since it's a 2x2 system there will be two loops L_1 and L_2. Where,

$$L_1 = g_1 / \pi_{11} \quad and \quad L_2 = g_2 / \pi_{22}^2 \tag{19}$$

$$\pi_{22}^2 = \pi_{22} - \frac{\pi_{21}\pi_{12}}{\pi_{11} + g_1} \tag{20}$$

Stability and tracking bounds are to be considered while designing g_1 and g_2

A. System Stability Evaluation Conditions

According to lemma deduced from 2 X 2 system QFT design process [9], sufficient conditions of 2X2 closed-loop system stability are:

i. $P^{-1} = [\pi_{ij}]$ Is stable

ii. $1 + g_1/\pi_{11}$ *and* $1 + g_2/\pi_{22}^2$ Do not have RHP zeros.

iii. G(s) is stable

Conditions (i) and (iii) can be satisfied by using minimum phase controller G(s). Condition (ii) is satisfied if equation (21) is true.

$$\left|1 + g_1/\pi_{11}\right|^{-1} \le 1.2 \ and \left|1 + g_2/\pi_{22}^2\right|^{-1} \le 1.2 \tag{21}$$

B. Controller Design

QFT based controllers are derived for the two loops while satisfying the specifications.A set of design frequencies is chosen as *ω=[0.1,1,3,5,10,100]*; using Matlab QFT Toolbox [10]; the plant templates at these frequencies are computed. The controller design proceeds using Nichols chart. The objective is to synthesize a controller, G(s), which satisfies the design specifications. The design objective is to apply dynamic compensation to the nominal open-loop transfer function, so that the performance bounds are satisfied at each frequency. Hence a dynamic compensator is required to change the shape of the open-loop frequency response. The controller for the first loop was finally designed as

$$g_1(s) = \frac{6.3 \times 10^5 (s + 1.56)(s + 2.6)}{(s + 175.6)(s + 121.7)} \tag{22}$$

Controller for the second loop;

$$g_2(s) = \frac{3.02 \times 10^4 (s + 10)(s + 11)}{(s + 217)(s + 205)} \tag{23}$$

Open loop frequency responses for both the loops with the controller are shown in "Fig. 5 and Fig. 6," respectively.

The controllers given in " (22) and (23)," are found to achieve the specification on upper and lower bounds for the closed loop frequency response in relation with $T_{RL}(j\omega)$ *and* $T_{RU}(j\omega)$. A prefilter is now designed, to achieve the required shape of the closed-loop frequency response to be within the range specified by the tracking bounds. The prefilter for loop 1 designed as

$$f_1(s) = \frac{10.3}{(s+10.3)(s+1)} \tag{24}$$

Prefilter for loop 2 is designed as

$$f_2(s) = \frac{10}{(s+2.5)(s+4)} \tag{25}$$

Step responses of the closed loop system with controller and prefilter are shown in Fig. 7, Fig. 8 and Fig. 9,". Results show that the time domain performance specifications are satisfied.

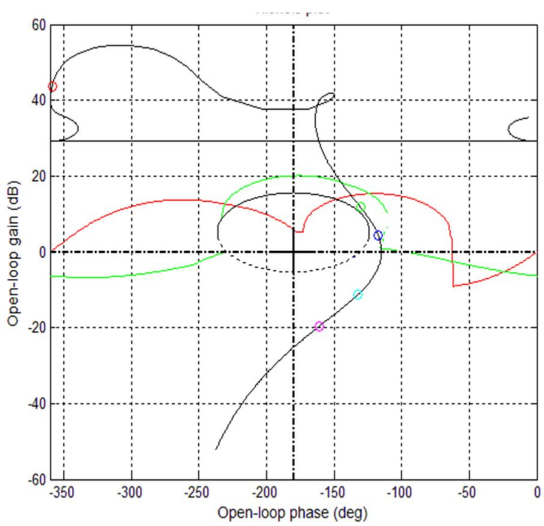


Figure 5. Open loop frequency response for loop 1

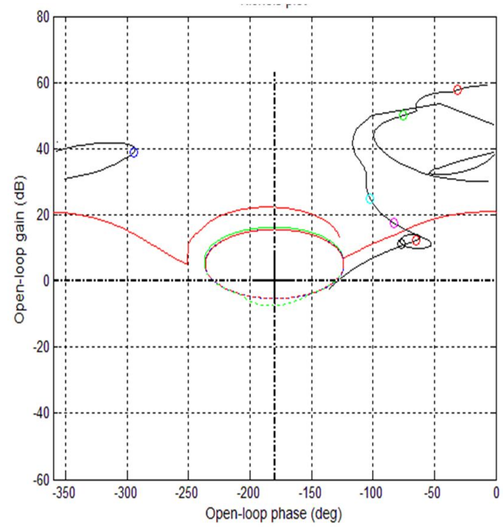


Figure 6. Open loop frequency response for loop 2

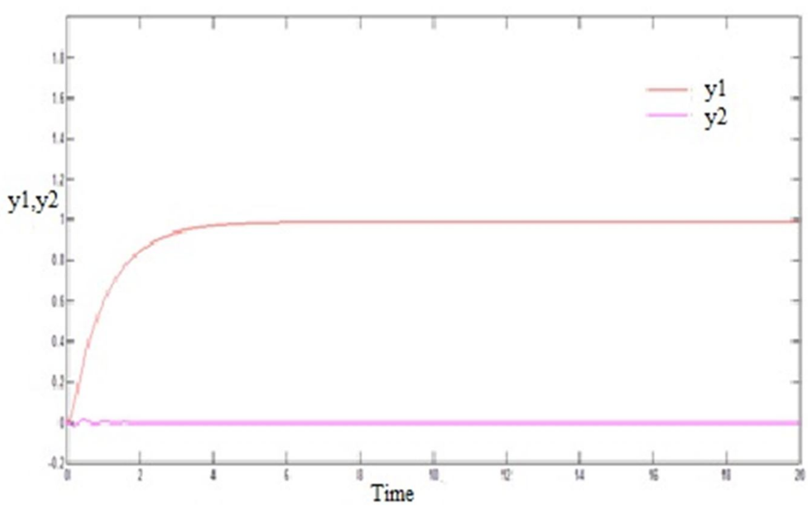


Figure 7. Closed loop responses y1 and y2 with unit step input at r1

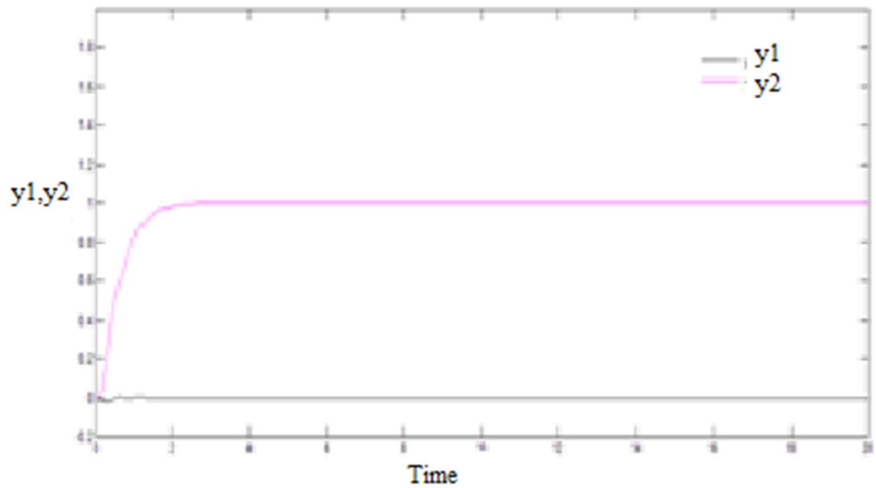


Figure 8. Closed loop responses y1 and y2 with unit step input at r2

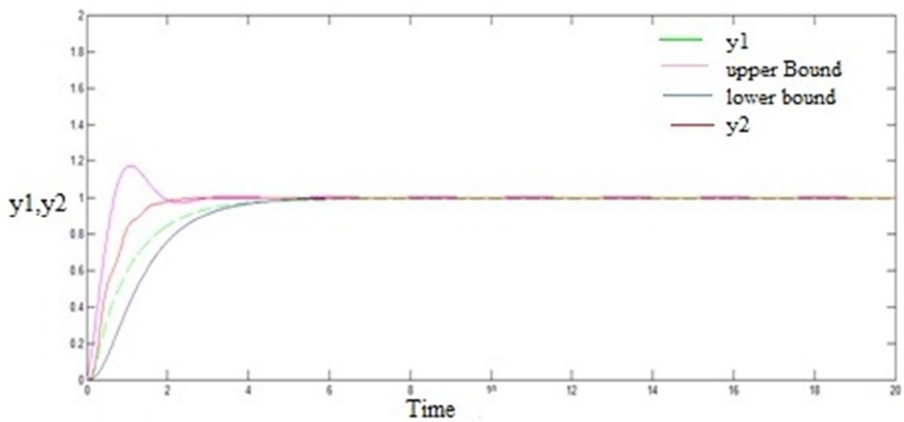


Figure 9. Closed loop responses y1 and y2 with unit step input at r1 and r2

V. CONCLUSION

This paper has presented the design of a diagonal robust controller based on MIMO QFT for TRMS. The design approach automatically takes into account the cross coupling effects inherent in the multivariable system. The system stability is guaranteed with the proposed MIMO QFT controller. The simulation results verifies the successful application of the methodology to the TRMS. The responses obtained with the proposed controller applied to the nonlinear model of the TRMS, establish that the controller – system combination can give accurate results during real time implementation.

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